Design of Non-Linear Systems using Fuzzy Logic Techniques

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Abstract

When investigating the behaviour of non-linear systems it is useful, at a fairly early stage, to be able to model them. Many simulation systems on the market will supply some standard non-linearities in their range of functions but often these are not sufficiently flexible to mimic the system under investigation. In this paper a straightforward method is presented which can mimic any non-linearities which possess real describing functions to a high degree of accuracy. The method utilises a fuzzy logic control system approach which uses Sugeno type one rule-bases and triangular fuzzy input sets which are just touching. The technique has proved to be easy to apply and to be a convenient method of actually designing non-linear effects into a system. Most discussions of non-linear modelling are content to simply find a method of predicting the ranges in which linear behaviour will still apply, or to predict where limit-cycles are likely to occur so that they can be avoided. In this paper a method of actually using the ability to create non-linearities to nullify the undesirable effcts which already exist in a system has been discussed.

Keywords- non-linearity; fuzzy sets; describing funtion; limit-cycle.

I. INTRODUCTION

The main thrust of this research has been to investigate the control of non-linear systems. That is to say, to investigate the control of the non-linear parts of those systems with the aim of understanding them sufficiently to be able to design non-linear efects Most of the early literature [1,2,5] deals with the characteristics and parameters of the various non-linearities which exist but beyond delineating their behaviour the emphasis has always been to mitigate the effects they have on the rest of the system. Even if that means operating the systems in 'safe' regions where the non-linear effects have little impact. This research has been concentrated on the non-linear effects themselves to see how they can be incorporated into the general control process.

Because the investigation is concerned with features of the non-linear effects that have largely been ignored it has been necessary, initially, to go back to the early work of the 1950s and, with the aid of more modern techniques, to build up a more comprehensive picture of how such effects operate and to see if there are any hidden patterns which might help in understanding them and which might eventually aid in the custom design of some of their more desirable features.

An early discovery was the usefulness of the describing function technique [2,3] for this work and a general approach was developed to derive these functions. The authors started by using this general approach and developed an algorithm [4] which enabled the rapid generation of real describing functions. In this paper the algorithm is briefly outlined and used to develop the describing functions for two non-linear systems, one of which possesses dead-zone plus saturation and therefore can exhibit a single limit-cycle and another which possesses four break-points and so can cause two nested limit-cycles to be produced.

The stability of these non-linear systems is discussed in terms of the cross-over points of their describing functions and the inverse-Nyquist diagrams of their respective linear sections. Kochenburger's Criterion [2,5] has then been used to predict the frequency and the magnitudes of the limit-cycles. In certain cases the describing function approach does not reliably predict all the limit-cycles which might exist [13]. However since our aim has been to manufacture non-linearities which have the properties which we desire the encountering of unexpected limit-cycles has not been a problem. Since only real describing functions have been considered, there can only be one frequency of oscillation at which any limit-cycle can occur for a given linear transfer function and whenever a system possesses more than one limit-cycle they must appear nested on the phase-plane diagram. The use of fuzzy-control techniques to create the non-linear effects is discussed in detail and a template explained which can then be used in their development. The simulations were then run using the SIMULINK package and the magnitudes and frequencies of any limit-cycles produced were measured. The predicted and simulated results are compared and this is followed by a general discussion and critique of the simulation technique itself.

II. DESCRIBING FUNCTION

A more up-to-date form of the usual harmonic approach to designing describing functions was used [2]. However, the theoretical formulation was confined to those non-linearities which produced real describing functions only, i.e.: to those non-linearities which did not possess memory. This simplification enabled an algorithm to be formulated [4] which considerably reduced the calculations necessary to derive individual describing functions and enabled the rapid investigation of large groups of non-linearities and the observation of patterns of behaviour not so easily seen when only a few samples are available.

A. The Algorithm

A basic assumption is that the non-linearity can be broken up into (n-1) linear sections with slopes $K_1 K_2 \cdots K_i \cdots K_n$ with sudden changes in slope (called break-points in this work) occurring at horizontal positions $P_1 P_2 \cdots P_i \cdots P_n$ (with a P_0 at the origin if necessary). Although the algorithm specifically deals with discrete cases it can easily be extended to deal with continuous functions.

Start:

If Coulomb friction or relay action is present the algorithm starts at stage one, otherwise is should be started at stage two.

Stage One:

(a) If Coulomb friction or relay action is present then $\dfrac{4Q}{\pi X}$ becomes the first term of the describing function, where Q is the value of the Coulomb friction term.

(b) If dead-zone is also present scale the term above by $\sqrt{\left[1-\left(\dfrac{P}{X}\right)^2\right]}$ where P is the dead-zone break-point.

Stage Two:

(a) If saturation is not present then K_n becomes the first term of the describing function. (K_n is the gain of the last stage of the non-linearity) or it is added to the result of stage one.

(b) If saturation is present then set $K_n = 0$.

Stage Three:

(a) If there are n breakpoints then add n terms of the form

$$\frac{2}{\pi}(K_{i-1} - K_i)\left(\sin^{-1}\left(\frac{P_i}{X}\right) + \left(\frac{P_i}{X}\right)\sqrt{\left[1 - \left(\frac{P_i}{X}\right)^2\right]}\right)$$

$$where\, i = 0 \rightarrow n. \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3a)$$

(b) If saturation is present then the last of the terms in stage 3(a) becomes

$$\frac{2}{\pi}K_{n-1}\left(\sin^{-1}\left(\frac{P_{n-1}}{X}\right) + \left(\frac{P_{n-1}}{X}\right)\sqrt{\left[1 - \left(\frac{P_{n-1}}{X}\right)^2\right]}\right)$$

Finish.

B. Stability

If the describing function is represented by $N(X, \omega)$ and the open-loop transfer function of a system is represented by $G(j\omega)$ then Kochenburger's Stability Criterion [4, 5] states that, in order for a system to remain stable, the locus $|G(j\omega)|$ must keep the entire locus – $|N(X, \omega)|^{-1}$ on the right; or the inverse locus $|G(j\omega)|^{-1}$ must keep the locus – $|N(X, \omega)|$ on the left (or it must completely enclose the whole of the locus). For this work the authors found that the inverse Nyquist approach was intuitive and considerably simplified calculations. Furthermore, since only systems with real, as opposed to complex, describing functions were being investigated, plots with real and imaginary axes were of little use. It was better to plot the magnitude of the describing functions against the magnitude of the input signal and superimpose on this the magnitude of the inverse Nyquist value at which it crossed the real axis. The position at which the descending describing function locus crosses the inverse Nyquist value then enables the magnitude of the limit-cycle to be determined, as shown in Fig. 1, and the frequency of the oscillation is that at which the Nyquist plot is entirely real.

III. SIMULATION OF THE NON-LINEAR EFFECTS

For the current research work the attraction of fuzzy control systems is that they are inherently non-linear and can themselves exhibit the range of features of classical non-linear systems. Also there has been some success in using the describing function method for the stability analysis of PI and PD fuzzy controllers [7, 9, 10]. Furthermore, fuzzy logic techniques provide a method of modifying the actual shapes of signals by design. This was the whole aim of the investigation and is something which it is not easy to do by other means.

The standard fuzzy controller design, Fig. 2, stems from the original one developed by Mamdani *et al.* [8]. He used a signal and its derivative as the inputs and this could be generalized to a group of input signals, each signal representing a different physical quantity. In our initial investigations we have only considered the non-linear behaviour of a single quantity and, since we are not considering non-linearities with memory at this stage , there is no need to look at its derivative. In these circumstances the system reduces to a single input set. It might be argued that Mamdani's fuzzification system is not needed in such a case but we have continued with the use of a pseudo-fuzzifier for three reasons: (i) the Matlab fuzzy toolbox provides the most convenient way in which to implement our non-linear control paradigm and if we should need to design a system which had greater precision then the fuzzy logic approach makes for very easy design using any general purpose programming language, (ii) when we come to look at non-linearites with complex describing functions two inputs are necessary, the original and its derivative, and then the full power of the fuzzy approach is needed.

Although the non-overlapping input sets adequately defined the linearities with sharp, clearly defined, non-linear break-points it is necessary to cope with the value at which the break-point occurs by making one end of each of the defined sets include that point. If this was not done then the existence of the undefined point caused sharp spikes to appear at the output. However, if the triangular inputs overlap slightly - by no more than about 5%, then the situation is produced in which one pseudo-linear range smoothly morphs into the next – which more accurately reflected real-life conditions, although the just-touching approach was used for the design work.

The basic Mamdani design [8] for a rule-base is a linguistically-based system and does not lend itself easily to mathematical manipulation. The Takagi-Sugeno-Kang [11] method is much more mathematically and, more importantly as far as this research is concerned, much more geometrically tractable than Mamdani's. In a zero-order Sugeno system a typical fuzzy rule has the form:

If input x is a fuzzy singleton in set A and input y is a fuzzy singleton in set B then output z = k where A and B are pre-defined input fuzzy sets and k is a constant. So all the output membership functions are singleton spikes. For the purposes of this research the zero-order system was not flexible enough. In a first-order Sugeno system the rules have the form:

If input x is a fuzzy singleton in set A and input y is a fuzzy singleon in set B then output $z = m*x+n*y+c$

where m, n and c are constants. The design of non-linearities with real describing functions only require a single input. So the rules for a first-order Sugeno system reduce to the form:

If input x is a fuzzy singleton in set A then output $z = m*x+c$.

IV. A TEMPLATE

In order that the non-linearities could be easily designed using the technique described, a template, Fig. 3, was devised in which all the important features of each design could be seen at a glance.

The template starts with the fuzzy, type one, triangular inputs which are just touching. The number of inputs required is determined by the number of break-points which are required together with the end-of-range values. Since linearities which are symmetrical about the origin are being investigated, the pattern of fuzzy sets will also be symmetrical about the origin. Also the fuzzy sets at each end define the range of inputs to which each system will be able to respond. The range must be chosen to be large enough to ensure that all signals of interest will be able to enter the system without being out-of-range and therefore not defined as far as the software is concerned. The result of such a scenaro would be that the output would be completely spurious and unrelated to the actual true state of affairs.

The outputs obey the Sugeno type 1 arrangement. The output between each pair of break-points is a straight line given by an equation of the form $y=Kx+C$ in which K represents the slope of the straight line and constant C is that value which satisfies the corresponding values of x and y at the start of that particular linear, or pseudo-linear, section. The rulebase is a one-to-one correlation between inputs and outputs taken in order.

Finally, the template shows the shape of the fuzzy rule-surface which, with this design arrangement, should correlate exactly with the shape of the non-linearity which would be seen if a unit ramp were input to this designed module in an open-loop arrangement.

V. THE SIMULATIONS

Two examples are presented (i) dead-zone plus saturation which can cause a single limit-cycle to be produced and (ii) a non-linearity which has four break-points (five pseudo-linear regions) and so can cause two nested limit-cycles. In each case the describing function was calculated using the algorithm already outlined and the non-linearity was placed in series with a third-order transfer function $G(s) = [s^3 + 5s^2 + 6s + 1]^{-1}$ and unit feedback then applied. Kochenburger's approach was applied together with the simplified graphical approach of Fig.1. This was then used to predict the frequency and amplitude of the limit-cycle oscillations. The fuzzy logic approach was then used to design the non-linearities and these were then incorporated into SIMULINK diagrams in series with the transfer function *G(s)* above, again with unity feedback. The actual SIMULINK oscillations were then compared with the predicted results from the describing function information.

A. Dead-zone plus saturation

This example is adapted from [12]. For the caculations using the algorithm the parameters are $n = 2, K_0 = 0, K_1 = K, K_2 = 0$. Stages two (b), three (a) & three (b) of the algorithm apply to give:

$$N = \frac{2K}{\pi}\left[\sin^{-1}\left(\frac{P_2}{X}\right) - \sin^{-1}\left(\frac{P_1}{X}\right) + \left(\frac{P_2}{X}\right)\sqrt{\left[1-\left(\frac{P_2}{X}\right)^2\right]} - \left(\frac{P_1}{X}\right)\sqrt{\left[1-\left(\frac{P_1}{X}\right)^2\right]}\right]$$

$$\text{when } X > P_2;\ N \text{ as for deadzone when } P_1 < X \le P_2 \text{ and } N = 0 \text{ when } X \le P_1$$

This result is shown graphically in Fig. 4.

For the non-linear design the template in Fig. 3a was used, with the input being defined over the range ±10. Five input sets were used: two trapezoidal NB and PB, and three triangular NS, ZE and PS; with breakpoints at -1.5, -0.5, 0.5 and 1.5. The one-to-one rulebase was as shown in the template, the outputs OPB, OPS, OZS, ONS and ONB being the straight lines defined in the ranges given by the breakpoints and the overall range values. The actual parameters of the output sets are shown in Fig. 3b. The resultant non-linearity was the rule-surface shown in Fig. 3a.

The calculated magnitude of the limit-cycle, from Fig. 4, is 1.79 ± 0.02. The actual magnitude of the limit cycle, from Fig. 5, is $1.78 \pm .03$. The calculated frequency of oscillation is 2.45 ± 0.001 rad/s and, from Fig. 5, the measured frequency of oscillation, with and without input overlap, is 2.44 ± 0.03 rad/s. The graphs showing the actual limit-cycle oscillations were indistinguishable when simulations were run with or without the input sets overlapping, provided the overlapping was small. The simulations broke down and produced spurious results if the overlapping was more than 5% These simulations were run several times and mean values of the output measurements in the non-overlapping cases calculated.

B. Non-linearity with four break-points (five slopes)

There were several possible combinations of pseudo-linear slopes which could have been used for this non-linearity; the one that was chosen exemplified some of the more important features of this type. For the algorithm the following relative values of the slopes were used: $K_0 > K_1, K_1 < K_2, K_2 > K_3$ *and* $K_3 < K_4$, which gave, as the describing function,

$$N = \frac{2}{\pi}\left[\begin{array}{l} K_4.\frac{\pi}{2} + (K_3 - K_4)\left[\sin^{-1}\left(\frac{P_4}{X}\right) + \left(\frac{P_4}{X}\right)\sqrt{\left[1-\left(\frac{P_4}{X}\right)^2\right]}\right] + \ldots \\ \quad (K_2 - K_3)\left[\sin^{-1}\left(\frac{P_3}{X}\right) + \left(\frac{P_3}{X}\right)\sqrt{\left[1-\left(\frac{P_3}{X}\right)^2\right]}\right] + \ldots \\ \quad (K_1 - K_2)\left[\sin^{-1}\left(\frac{P_2}{X}\right) + \left(\frac{P_2}{X}\right)\sqrt{\left[1-\left(\frac{P_2}{X}\right)^2\right]}\right] + \ldots \\ \quad (K_0 - K_1)\left[\sin^{-1}\left(\frac{P_1}{X}\right) + \left(\frac{P_1}{X}\right)\sqrt{\left[1-\left(\frac{P_1}{X}\right)^2\right]}\right] \end{array}\right]$$

This characteristic presents a slightly different situation to the previous case. Now there are two positions at which a limit-cycle may occur, Fig. 6, the first at 1.8 ± 0.05 and the second at 5.05 ± 0.05. However, looking at the right of the graph, a critical point is marked at 9.85 ± 0.3. In this case it is possible for a rising value of the describing function to cross the inverse Nyquist locus a second time. This holds out the potential for instability if the input signal rises higher than this critical value. There is also a (less) critical point at about 3.41. If the input signal is higher than this value the system enters a region in which the gain is greater than unity. The result will be that as the error signal is swept around the loop its value will continue to increase until the second limit-cycle position is reached. So the input signal does not have to reach a value of 5.08 to initiate the second limit-cycle oscillation; all that is necessary is that it is higher than the first critical point and it will automatically be amplified to the second limit-cycle value.

In order to design the fuzzy equivalent of this describing function the template in Fig. 3 was modified to include nine input and nine output sets. When used in simulation with third-order transfer function as in the first example, the graphical output shown in Fig. 7 was produced. From this graph the measured frequency of the limit-cycle oscillation was 2.41 ± 0.07 rad/s which compared with the calculated frequency of 2.45 ± 0.001 rad/s. Two limit-cycles were present, the actual magnitude of the lower being 1.85 ± 0.26 and that of the higher 4.90 ± 0.26. To see if the second critical point existed, the simulation was run with increasingly higher inputs and it was found that above 9.70 ± 0.26 the output became unstable with the amplitude rising uncontrollably. A lower critical point was also seen but its position appeared to be more variable at 4.1 ± 0.5.

VI. CONCLUDING REMARKS

The use of this simulation enabled discontinuous non-linearities which have straight sections between break points to be easily designed. Further it was found that the technique could be extended to continuous non-linearities. The authors are not aware of any other technique which allows non-linear system to be designed in such an easy and straightword manner. Although examples of the design of only two representative non-linearities have been presented, the technique has been applied to a considerable range of real and simulated non-linear systems and the simulated results have consistently agreed closely with real-world situations. Furthermore, the technique has allowed a sufficiently large range of non-linearities to be rapidly developed for it to be possible to identify patterns between them which were not previously obvious or have not been reported in the literature – work which has stimulated further research. Although the technique has only been been demonstrated in this paper for systems which have real describing functions it can quite easily be applied to non-linear systems which possess memory and therefore have complex describing functions.

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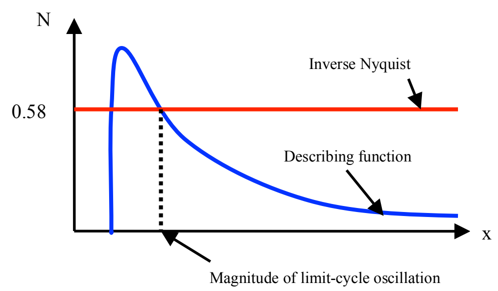


Fig. 1: A real describing function locus with an inverse Nyquist magnitude superimposed on it.

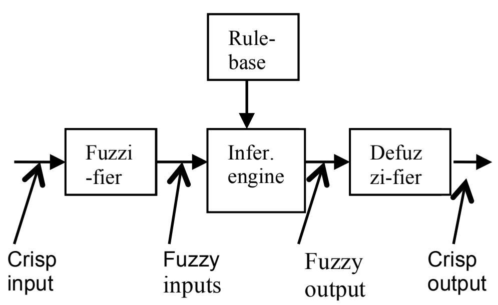


Fig.: 2: A typical fuzzy control arrangement

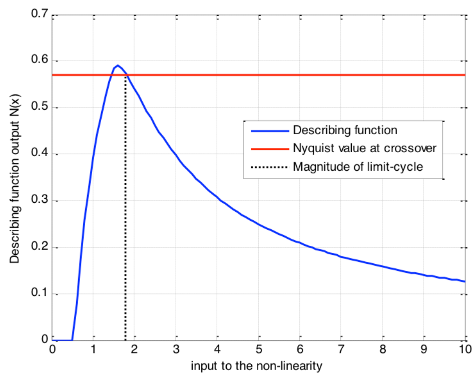


Fig. 4: Describing function of dead-zone plus saturation

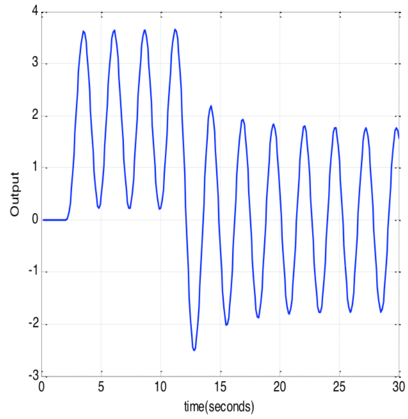
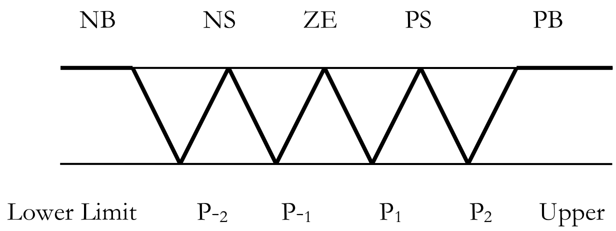


Fig.6: Limit-cycle oscillation with saturation and dead-zone

INPUT



OUTPUT

OPB	[K_2 C_2]	$Z= K_2 X + C_2$
OPS	[K_1 C_1]	$Z= K_1X + C_1$
OZS	[K_0 C_0]	$Z= K_0 X + C_0$
ONS	[K_{-1} C_{-1}]	$Z= K_{-1} X + C_{-1}$
ONB	[K_{-2} C_{-2}]	$Z= K_{-2}X + C_{-2}$

RULEBASE

If input is NB then output is ONB
If input is NS then output is ONS
If input is ZE then output is OZE
If input is PS then output is OPS
If input is PB then output is OPB

RULESURFACE

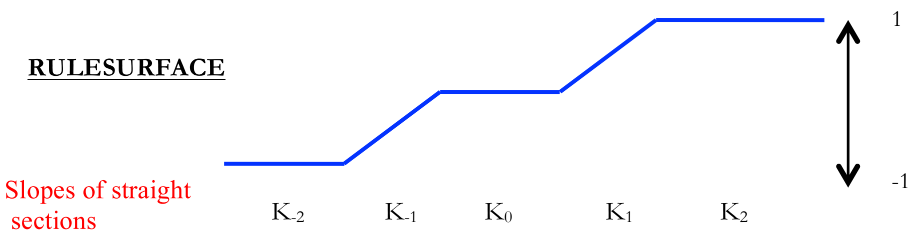


Fig.3a: An exemplar template for the design of a fuzzy non-linear function, (dead-zone plus saturation in this case).

OPB	[0, 1]	z = 1
OPS	[1, -0.5]	z = x – 0.5
OZS	[0, 0]	z = 0
ONS	[1, 0.5]	z = x +0.5
ONB	[0, -1]	z = -1

Fig.3b: The output sets for dead-zone plus saturation

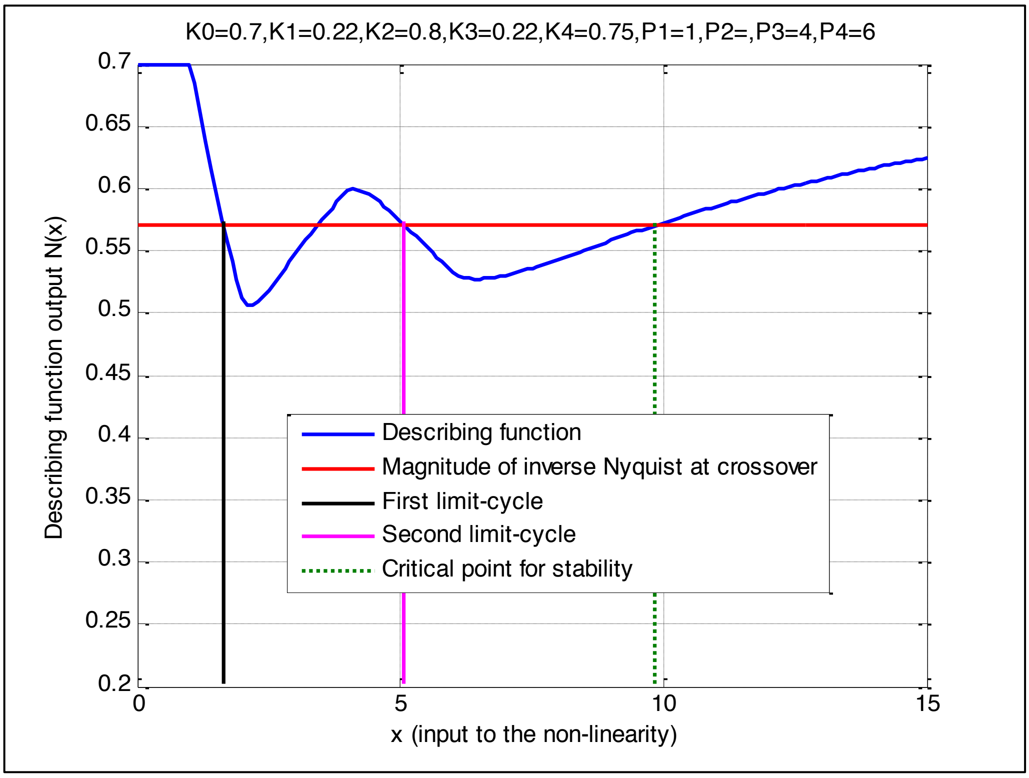


Fig. 6: Describing function for the four-breakpoint non-linearity

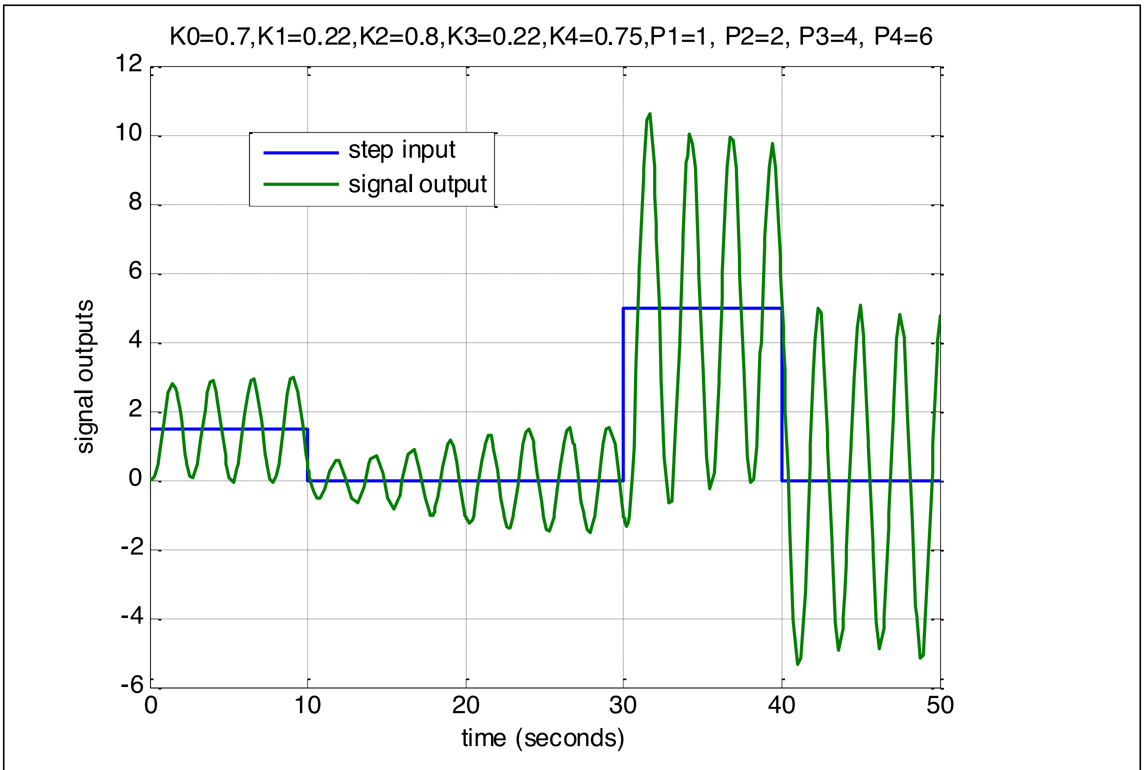


Fig. 7: Fuzzy four-breakpoint response showing two limit-cycles