Sliding Mode Control for a Boost Converter Based on a Current Source Load

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Abstract

In this paper, a model for a boost converter is proposed that models the load as a current source. Unlike the conventional model that models the load as a resistor, the use of a current source makes the model simpler without reducing performance. Based around the proposed model, a sliding mode controller is designed to control a boost converter. The control is inspired by the isoclines of the 'on' and the 'off' models. The stability of the controlled system is theoretically proven and then validated by simulation using SimuLink PSpice (SLPS). The control strategy/model can be adapted for use on most DC-DC converters.

1 Introduction

Personal power packs with on-board renewable sources are being developed to minimise the weight of energy storage devices that a soldier has to carry. Such packs require a power management system for transferring power between sources, storages and loads. Power management systems, for the scale required for the personal power packs, use DC-DC converters. The important factors to be taken into consideration while developing and optimising control for a converter are regulating the inputs/outputs at the reference voltages, transferring the required amount of current between inputs/outputs and maximizing energy efficiency.

The boost converter is the simplest design of DC-DC converter that creates a positive output voltage that is greater than the input voltage. The boost converter is bilinear. In the model the control input is multiplied by the system states in the differential equations [1]. This means that a purely linear control strategy is not suitable because a linear control based around a linearised model would only have local stability and not global stability [1]. There have been proposed control strategies that consist of a current control loop inside a voltage control loop [2], [3], [4]. A boost converter will generally be designed to have the inductor current react faster than the output voltage. This is because only the output capacitor needs to be regulated at a certain value. The current loop inside voltage loop control strategy regulates the voltage by altering the reference of the inductor current control. The current loop inside the voltage loop allows conventional linear control to be used by splitting the model into two linear first order models. However, because the system is made up of two controllers it is difficult to optimise performance/stability of the system over a wide range. Any analogue control strategy (such as linear control) must be implemented using pulse-width-modulation (PWM) in order to be used on a switch mode converter. The optimisation of the control is not just a question of the regulation of the output at the reference. The energy efficiency of the converter also needs to be maximised. A greater switching frequency results in closer potential regulation of the output but it also results in greater switching losses. Control strategies with a constant switching frequency are not the best option for a converter that operates across a wide range. This is because a compromise in switching frequency is required in order to maximise performance.

Sliding mode control (SMC) is robust [5] and due to its on/off nature it can be implemented on limited input systems such as DC-DC converters without the need for any modification of the controller output.

The organisation of this paper is as follows. Section 2 presents further background into the boost converter, conventional modelling of a boost converter, conventional control of a boost converter, and further background on sliding mode control. Section 3 presents the proposed boost converter model. Section 4 presents the proposed control strategy derived from the proposed model and proves the stability. Section 5 presents simulation results of the proposed control strategy. Finally section 6 is the conclusion.

2 Background

2.1 Boost Converter

The boost converter converts an input DC voltage into a higher output DC voltage (figure 1). During operation the transistor is turned on/off continuously thereby altering the effective circuit. The setup causes more current to pass through the input than the output. Assuming that there are no losses, there must therefore be a higher voltage at the output than at the input. Equation 1 shows the relationship between the duty (‘d’) of the switch and the input and output voltages.

$$\frac{v_{out}}{v_{in}} = \frac{1}{1-d}\,, d \in [0, 1] \tag{1}$$

2.2 Conventional way to model a boost converter for control purposes

Conventionally the load is modeled as a resistance. So long as the inductor current is positive and the output current is positive (both of these are generally true when the converter is in operation), the diode will be on when the transistor is off and off when

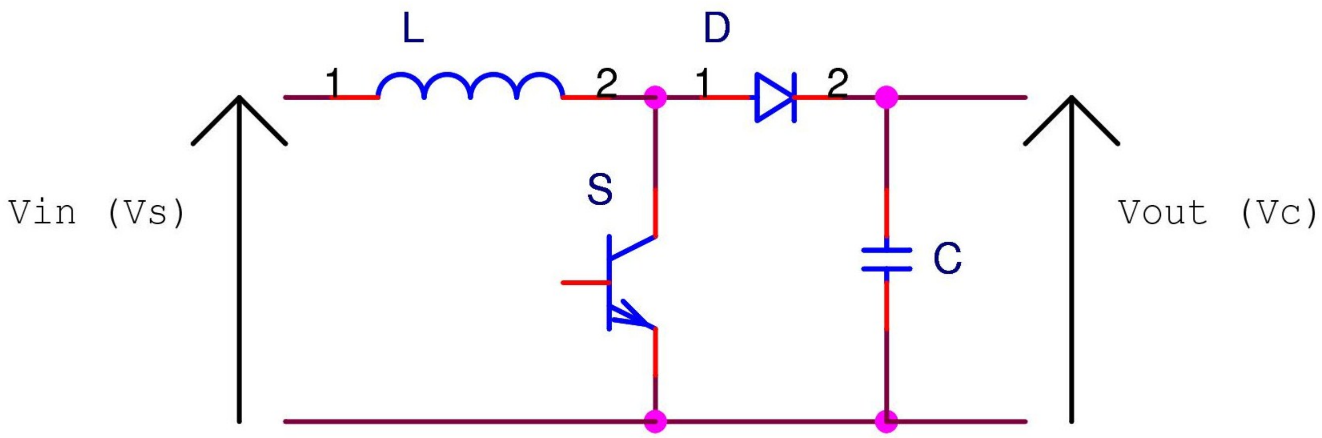

Figure 1: Boost Converter

the transistor is on. This creates equivalent circuits for when the transistor is on and for when it is off. Models for the 'on' and 'off' modes can then be derived. This is done by calculating the voltage across the inductor and the current passing through the capacitor when the inductor is represented as a current source and the capacitor as a voltage source. Thus differential equations for the inductor current and capacitor voltage can be derived as a function of the state variables (inductor current and capacitor voltage) and the external inputs (input voltage and output resistance). Because there are only two modes, they can be combined to create one model. The 'on' model is multiplied by the input, 'u', and the 'off' model is multiplied by '(u-1)'. The two modes are then added together to create the common model. The model produced after simplification is bilinear and is shown in equations 2 to 4.

$$\dot{i}_L = \frac{v_S - (1-u)v_C}{L} \tag{2}$$

$$\dot{v}_{out} = \frac{(1-u)i_L - v_C/R}{C} \tag{3}$$

$$u \in (0,1) \tag{4}$$

In the case that the switch is off ('u=0') the model indicates an underdamped second order system. However, when the switch is on ('u=1') the model indicates that the system is decoupled into an integrator and a stable first order system. The way that the model is bilinear means that standard linear control strategies do not work when attempting to regulate the output voltage. A linear controller made up of two control loops (voltage and current) that does not measure output current would require an integral control component to regulate a load with a variable resistance (e.g. a load being switched on/off). It would take time for the integral component of the control to adapt to the change. This would make such a controller suboptimal if the load resistance were to change quickly. In order to make a purely continuous model control strategy sensitive to varying amounts of current being extracted, the model would need to be continually updated.

2.3 Sliding Mode Control

Sliding mode control (SMC) is a form of control that is robust. SMC is designed to attract the states variables onto a sliding surface. Once the system is on the surface, the system follows the path of the sliding surface to the reference. The control output contains a step signal as the system crosses the surface. The resultant chattering produces robustness to model error.

3 Modelling the Boost Converter with a Current Source as the Load

The equivalent circuit is produced by modelling the load as an external input and representing it as a current source. The model is derived using the same method as was used for the conventional resistive load model. The model of the system is shown in the following equations. The model produced is simpler than that for the resistive load. Under the assumption that the input voltage and the output current are constants, the system is an undamped resonator when the switch is off ('u=0'), and two decoupled integrators when the switch is on ('u=1').

$$\dot{i}_L = \frac{v_S - v_C(1-u)}{L} \tag{5}$$

$$\dot{v}_C = \frac{i_L(1-u) - i_{out}}{C} \tag{6}$$

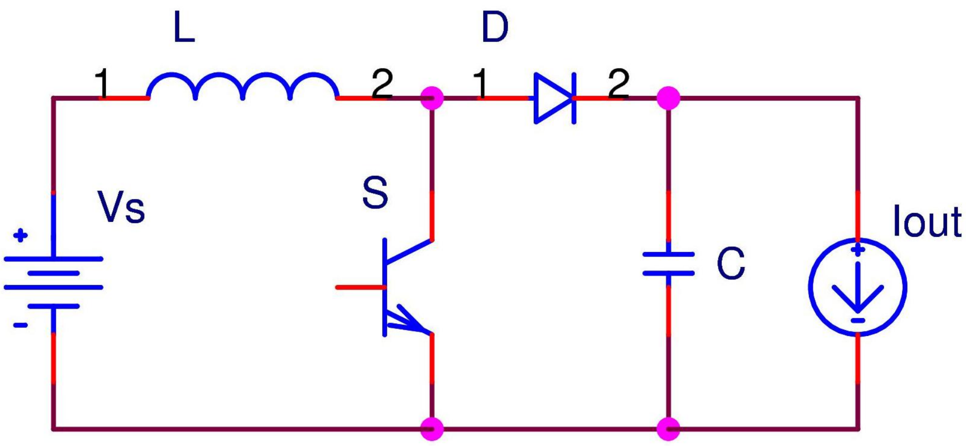

Figure 2: Boost Converter with Current Source as Load

4 Proposed Sliding Mode Control Strategy

The control strategy was inspired by analysing the isoclines of the 'on' and 'off' system and visualising the manifold that would produce global stability and would mean that the path from any point in the phase plain to the reference would only require one section of 'on' mode and one section of 'off' mode.

$$\mathbf{x} = \begin{bmatrix} i_L \\ v_C \end{bmatrix} \tag{7}$$

The '$\mathbf{x_{ref}}$' represents the equilibrium point that satisfies the voltage reference, 'v_{ref}'. The reference current 'i_{ref}' is calculated by assuming that the converter has 100% energy efficiency. The formula is shown as equation 9.

$$\mathbf{x_{ref}} = \begin{bmatrix} i_{ref} \\ v_{ref} \end{bmatrix} \tag{8}$$

$$i_{ref} = \frac{i_{out} v_{ref}}{v_S} \tag{9}$$

The manifold is produced by reversing the route of the states variables from '$\mathbf{x_{ref}}$'. A path is created representing how the state variables reach '$\mathbf{x_{ref}}$' in the 'on' mode, and it forms the sliding function 's_1'.

$$s_1 = v_C - v_{ref} - \frac{i_{out} L (i_{ref} - i_L)}{v_S C} \tag{10}$$

The 'off' path produces an elliptical section of the manifold, and it forms the sliding function 's_0'.

$$s_0 = (\mathbf{x} - \mathbf{x_{offEq}})^T \mathbf{m} (\mathbf{x} - \mathbf{x_{offEq}}) - (\mathbf{x_{ref}} - \mathbf{x_{offEq}})^T \mathbf{m} (\mathbf{x_{ref}} - \mathbf{x_{offEq}}) \tag{11}$$

To determine if the state variables are within the bounds of the ellipse, the domain is altered by replacing 'i_L' with a scale factor of itself. The effect is to convert the ellipse into a circle. The modulus between the state variables, '$\mathbf{x}$', and the centre of the circle (the equilibrium point of the 'off' mode), '$\mathbf{x_{offEq}}$' can be compared with the radius of the circle. The radius is calculated as the modulus of the difference between '$\mathbf{x_{ref}}$' and '$\mathbf{x_{offEq}}$'. The matrix '$\mathbf{m}$' conducts a state transform. The element 'm_{11}' is the square of the factor required for the state transform. This is used because the vectors are processed through a dot product that is shown as a matrix multiplication in the equation. The value 'm_{11}' is calculated as the square of the amplitude of the voltage relative to the square of the amplitude of the current. It is derived from the transform function shown in equation 15.

$$\mathbf{x_{offEq}} = \begin{bmatrix} i_{out} \\ v_S \end{bmatrix} \tag{12}$$

$$\mathbf{m} = \begin{pmatrix} m_{11} & 0 \\ 0 & 1 \end{pmatrix} \tag{13}$$

$$m_{11} = \frac{{v_S}^2 + \frac{L{i_{out}}^2}{C}}{\frac{C{v_S}^2}{L} + {i_{out}}^2} \tag{14}$$

$$\begin{bmatrix} i_L(s) \\ v_c(s) \end{bmatrix} = \frac{\begin{pmatrix} \frac{1}{LC} & \frac{s}{L} \\ \frac{-s}{C} & \frac{1}{LC} \end{pmatrix}}{s^2 + \frac{1}{LC}} \begin{bmatrix} i_{out}(s) \\ v_s(s) \end{bmatrix} \tag{15}$$

The surface stops following the ‘off’ path when the states are opposite the reference. The further continuation of the manifold is produced using an ‘on’ path, thereby creating a straight line that is a tangent of the ellipse and is in parallel with the opposite sliding surface. The sliding function is shown as equation ‘16’.

$$s_2 = 2v_s - v_{ref} + \frac{i_{out}L(2i_{out} - i_{ref} - i_L)}{v_s C} - v_C \tag{16}$$

A line drawn from ‘$\mathbf{x_{ref}}$’ to the opposite side of the ellipse marks the ends of all the sliding surfaces. This forms a trapezium-shaped zone with the straight sliding surfaces. The line is represented by equation 17.

$$s_3 = v_S + \frac{(v_{ref} - v_S)(i_L - i_{out})}{i_{ref} - i_{out}} - v_C \tag{17}$$

An example of the manifold and isoclines is shown in figure 3. Equation 18 determines whether the state variables are within bounds the elliptical zone or the trapezium-shaped zone, if so, the output ‘u’ is set to switch the transistor on.

$$u = \left\{ \begin{array}{l} 1, s_0 < 0 \text{ or } \; [(s_1 < 0)\,\&\,(s_2 < 0)\,\&\,(s_3 < 0)] \\ 0, \text{otherwise} \end{array} \right\} \tag{18}$$

The control will continue to work when the current becomes zero in discontinuous conduction mode, because the voltage will decrease, the current will remain at zero, and the state variables will converge on the target sliding surface and continue as normal from there.

The method used for developing this control strategy can be used on a variety of bilinear and bidirectional converters (e.g. a Buck Converter used to charge a constant voltage with a generator as an input that needs to be regulated at the maximum power point (MPP), a Buck-Boost Bidirectional Converter).

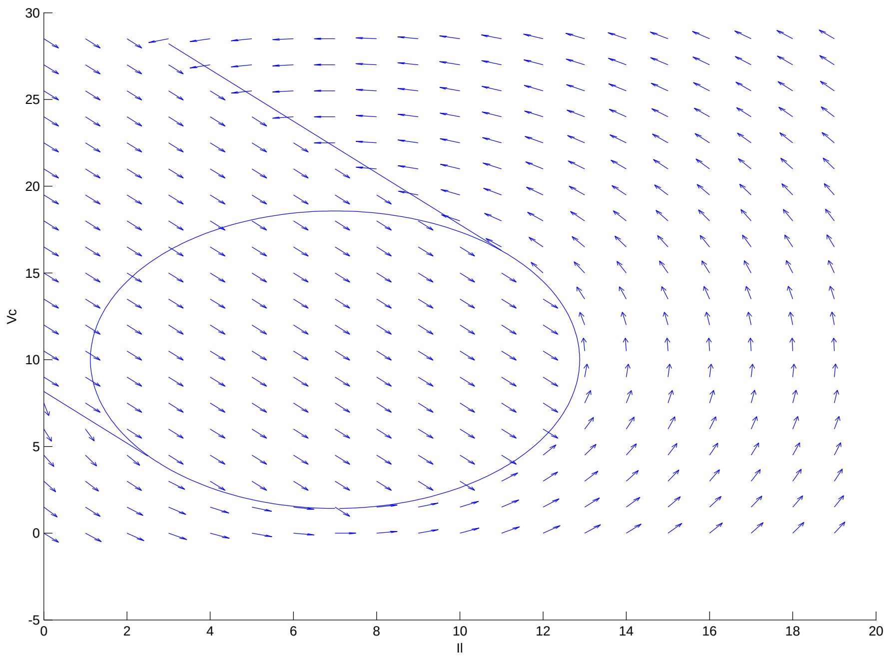

Figure 3: Isoclines of current load model under the proposed SMC

5 Simulation Results

This section will show simulation results for the system and compare the control with a simple single step model predictive based sliding mode control.

5.1 Setup

The tests were conducted using the SLPS interface between PSpice and MATLAB. SLPS produces SimuLink blocks that represent the PSpice model. The SLPS block inputs are the voltage source, the load current and the MOSFET gate voltage. To replicate a varying load, the output current was set up as it would be if a load of a constant or varying resistance was attached. The SLPS block was used to test whether the proposed control strategy has the required robustness to account for model error. An example of model error is the difference between the modeled capacitance of the MOSFET (zero) and the value produced by PSpice. The control was implemented using a level-2 S-function block and an m-file. The reference was also set to vary during the test in order to analyse the transience.

5.2 Testing the sliding mode control system with a constant sample frequency

In order to regulate the switching losses, a potential maximum switching frequency of 500 kHz was set by setting the control sample time at 1 us. The results appeared to show a

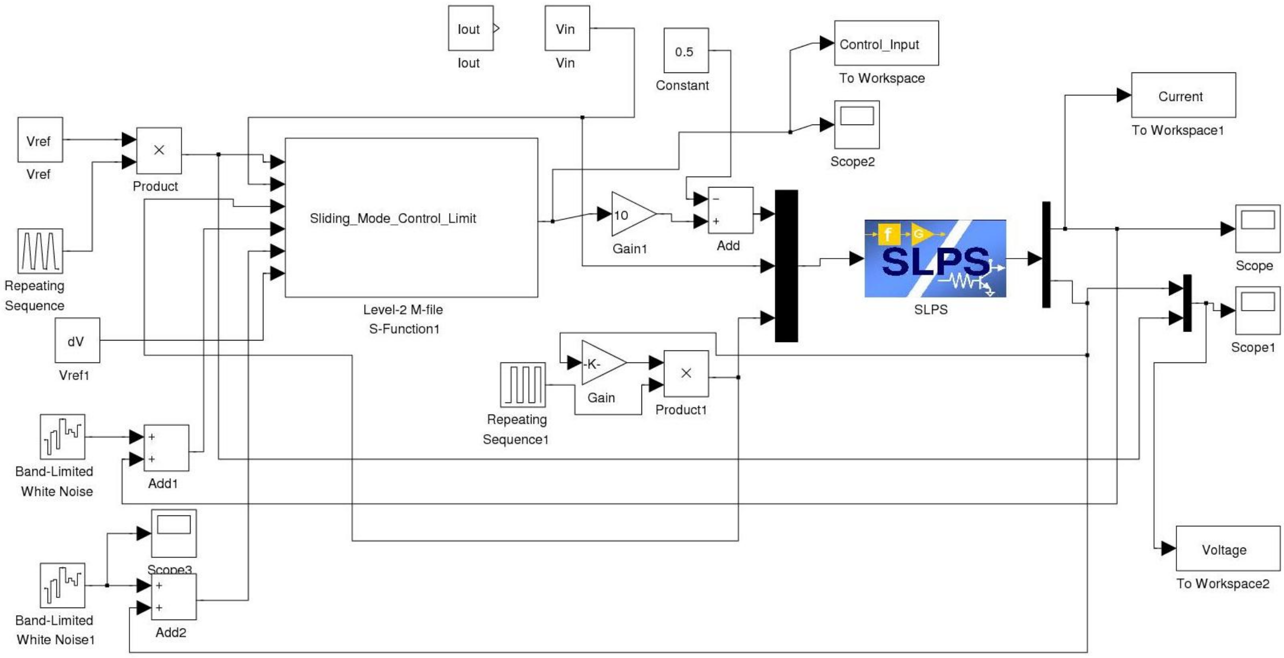

Figure 4: Simulink Model

perfect regulation even though the model used to design the control was far simpler than the model that was used by PSpice during the simulations. The results indicated that the control was working as it was intended to (regulating the output at the reference by following the manifold).

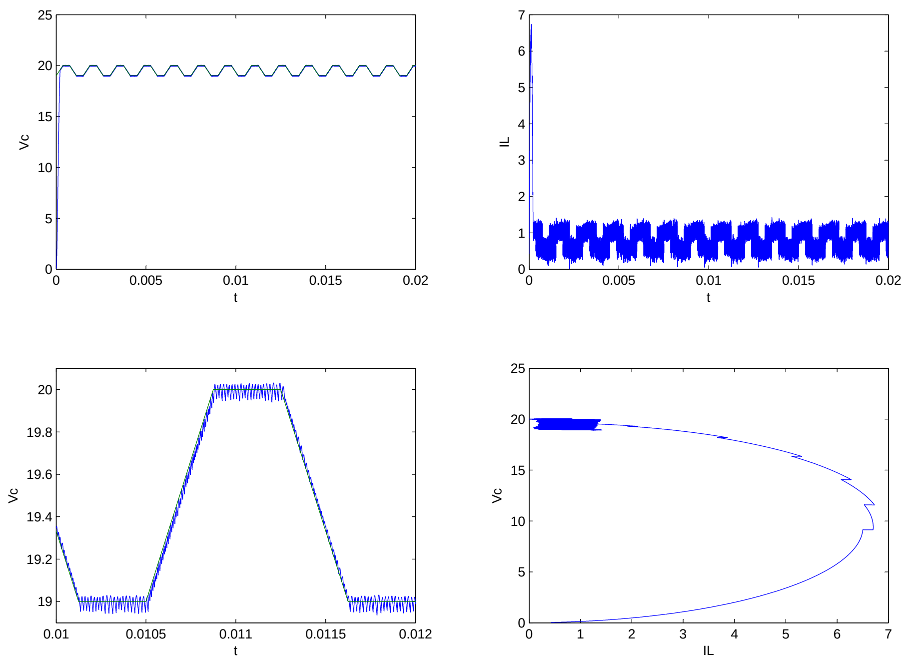

Figure 5: Results of test of Control System with PSpice model and accurate capacitor and inductor values and constant load resistance

5.3 Testing the sliding mode control with a varying load

The resistance of the load was setup in the SimuLink simulation to vary between two quantities. The Variation was in the form of a square wave and the variance was a factor of two (Load resistance is half of previous test half of the time). When run the control did exactly what it should have. The current reference increased and the control took the system directly to the new manifold and then to the reference.

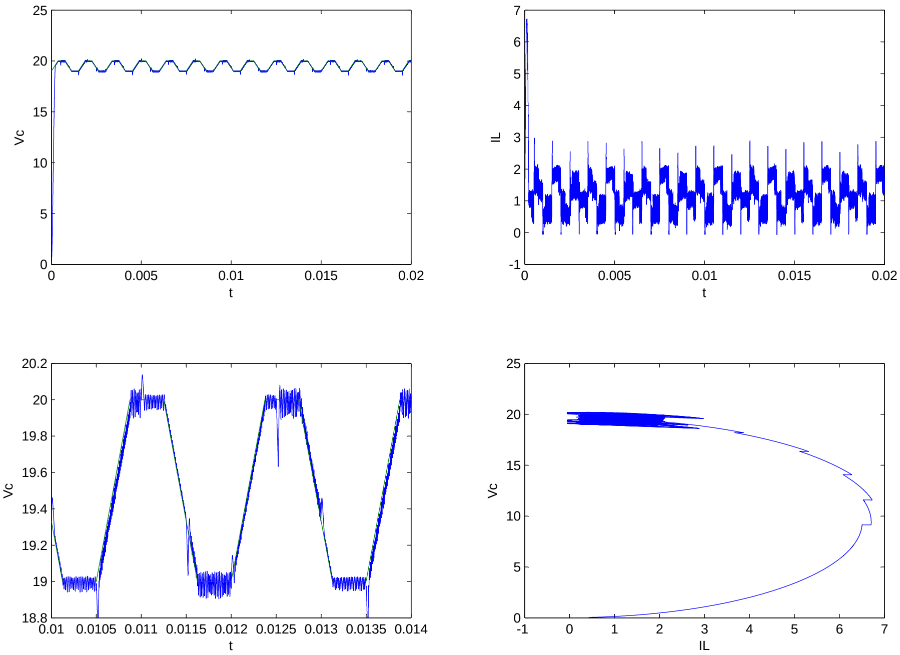

Figure 6: Results of test of Control System with PSpice Model and Accurate Capacitor and Inductor Values and Varying Load Resistance

5.4 Testing the Sliding Mode Control with inaccurate system parameters

One of the known advantages of sliding mode control is the ability to track the reference when the model is inaccurate. The inductance and capacitance used by the controller to calculate the control were set with a 25 % error (inductance reduced and capacitance increased). When tested, this appeared to have a very minimal affect on the tracking of the reference but the chattering had increased. The system was then tested with the opposite model error (increased inductance and decreased capacitance). There was a very minimal difference in performance between the response to this and the accurate model. Increasing both the inductance and the capacitance was not tested because if both were increased by the same factor, the manifold would remain unchanged.

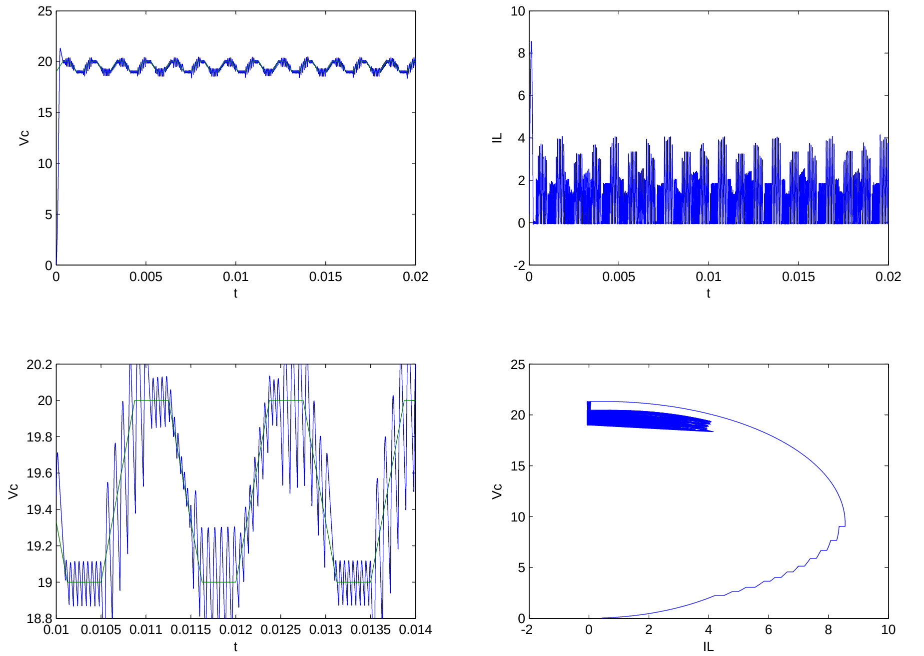

Figure 7: Results of test of Control System with PSpice Model and underestimated inductor and overestimated capacitor Values and Varying Load Resistance

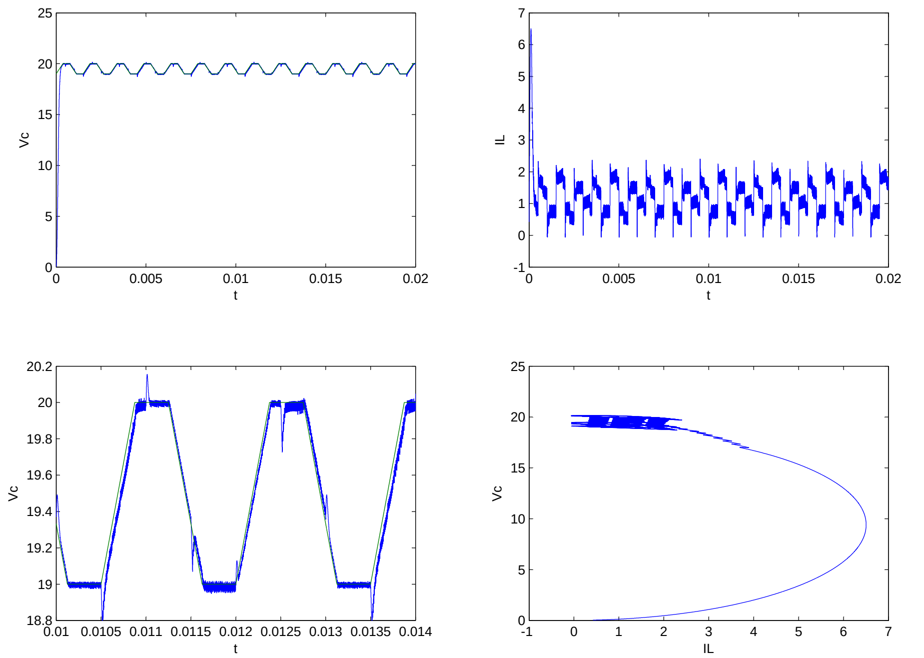

Figure 8: Results of test of Control System with PSpice Model and overestimated inductor and underestimated capacitor Values and Varying Load Resistance

5.5 Comparing the results with that of a basic controller

A model-predictive based sliding mode control was produced to demonstrate that there is no simple solution to the control problem. The function of the control is to minimise a sum of square errors function for the next sample point. The error function is weighted and takes account of both voltage error and current error. It needs to consider current in order to be able to reach the reference. Otherwise the negative short term effect that the input has on the voltage will lead to the current not increasing enough to allow the voltage to increase enough. The weights were optimised using manual trial and improvement. The test of the decision as to whether the switch should be 'on' or 'off' was converted into the following inequality:

$$cost_{on} < cost_{off} \tag{19}$$

There appeared to be no difference between the model error test and accurate model test. One reason for this is that the weight of the current in the cost function needed to be so high to account for the bilinearity (in order to make the output voltage reach the reference) that the system was mainly using current control and so the inductance and capacitance values would have very little effect on the choice of output. The results show that the MPC based SMC does not converge as quickly as the proposed SMC. This is because the MPC only looks at the next step and consequently takes a route that is shorter but slower. A more sophisticated MPC might be able to get pass these limitations but this demonstration was set up to prove that there is not a simple solution to the problem of controlling a boost converter.

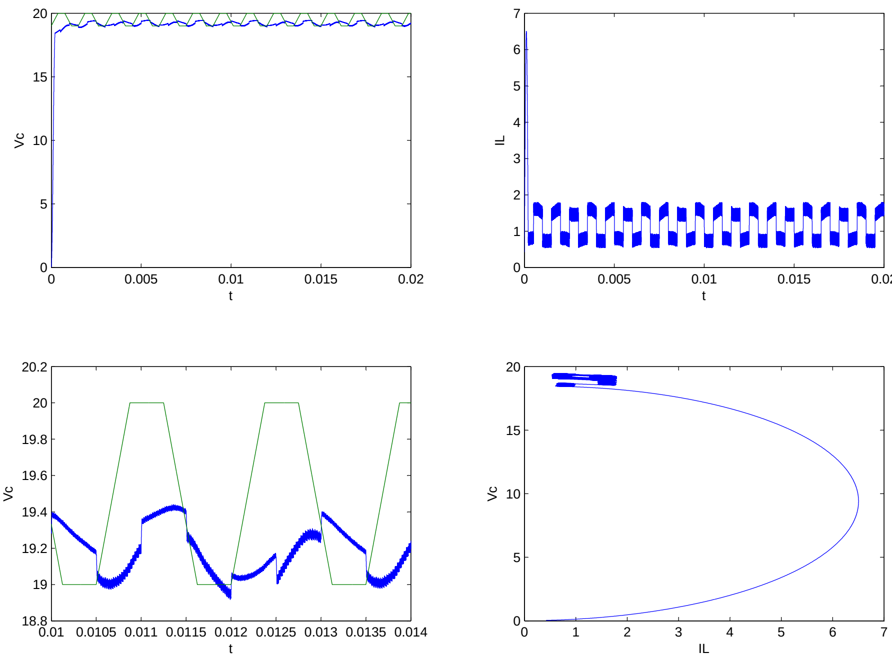

Figure 9: Results of MPC Based SMC with Correct Model Values and Varying Load

6 Conclusion

A sliding mode control system that is based around isoclines has been created and successfully tested for a boost converter. It has the ability to adapt to incorrect model parameters and a varying load. The strategy could be modified to work on any DC-DC converter where an input/output voltage having dynamics (e.g. capacitor) makes the system bilinear (e.g. Photovoltaic powered buck converter based battery charger). The results showed that system robustness was not effected and perhaps increased by over estimating the inductor and underestimating the capacitor. This indicates that the control has robust stability to model error so long as the inductance is overestimated and the capacitance is underestimated. This is because it alters the equivalent inputs across the surfaces (that are close to the reference) to be between zero and one, but it might also have a negative effect on the speed at which the system converges to the reference whilst on the surface. Therefore a compromise between performance and stability could be produced. To be used in practise a method for measuring output currents would need to be implemented. Some options would be to measure the current manually, to use a non-linear Kalman filter or to use a combination of the two. Further research could also be done into modifying the sliding surfaces in order to optimise them.

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